

SOME ANNIHILATOR IDEALS IN SKEW HURWITZ SERIES RINGS

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Abstract. A ring R has right (left) property (A) if for every finitely generated two-sided ideal $I \subseteq Z_l(R)$ ($I \subseteq Z_r(R)$), there exists nonzero $u \in R$ ($v \in R$) such that $Iu = 0$ ($vI = 0$). In this article, we establish a relationship between a ring with property (A) and its skew Hurwitz series ring (HR, ω) , where ω is an endomorphism of R . Also some properties of strongly right AB ring for skew Hurwitz series rings are studied.

Keywords: ring with right property (A); skew Hurwitz series ring; ω -compatible ring

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1. INTRODUCTION

Throughout this article, R denotes an associative ring with identity. For any subset P of a ring R , $r_R(P)$ denotes the right annihilator of P in R . According to Kaplansky [23] if R is a commutative Noetherian ring, then the annihilator of an ideal I consisting entirely of zero-divisors is nonzero. This result is not true for the non-Noetherian ring, even if the ideal I is finitely generated. Furthermore, Huckaba and Keller [18] introduced the concept of a commutative ring with property (A). A commutative ring R has property (A) if every finitely generated ideal of R consisting entirely of zero-divisors is a nonzero annihilator. This class of rings is quite large and contains some well known classes of rings. For example: the class of rings whose prime ideals are maximal [14], the polynomial rings $R[x]$, the class of Noetherian rings [22] and the rings whose classical rings of quotients are von-Neumann regular. Initially, Quentel [32] studied the concept of rings with property (A). He used the term condition (C) instead of property (A). Using property (A), Hinkle and Huckaba [15] generalized the concept of Kronecker function rings from integral domain to rings with zero-divisors. The class of commutative rings with property (A) has been studied by several authors, see [4], [9], [10], [14], [15], [17], [18], [28], [40]. In

2007, Hong et al. [17] extended the concept of commutative rings with property (A) to noncommutative rings. They defined that a ring R has right (left) property (A) for every finitely generated two-sided ideal $I \subseteq Z_l(R)$ ($I \subseteq Z_r(R)$) if there exists a nonzero $u \in R$ ($v \in R$) such that $Iu = 0$ ($vI = 0$). A ring R is said to have property (A) if R has right and left property (A). They proved some important properties of a ring with right property (A) and established the following cases:

- (1) If R is a reduced ring with finitely many prime ideals, then R has property (A).
- (2) If R is a reversible ring and every prime ideal of R is maximal, then R has property (A).
- (3) If R is a biregular ring, then R has property (A).

Moreover, they studied several extensions of rings with property (A) including matrix rings, polynomial rings $R[x]$ and classical quotient rings. They also raised following questions:

- (1) Does the power series ring $R[[x]]$ over a commutative ring R have property (A)?
- (2) If a ring R has right property (A), then does the power series ring $R[[x]]$ over R have right property (A)?

Hashemi et al. [10] studied the above questions and gave a negative answer to question 2 from [10] and showed that there exists a ring R which has right property (A), while the power series ring $R[[x]]$ does not have right property (A). They answered question 2 positively when R was reversible and Noetherian. They proved that if R is reversible and Noetherian, then $R[[x]]$ has property (A). Further, Hashemi et al. [9] proved that if R is a right Noetherian right duo and an ω -compatible ring, then the skew power series ring $R[[x; \omega]]$ has right property (A). Moreover, they gave the answer of question 1 in [9] if R is commutative Noetherian. However, they showed in [9], Example 2.12 that there exists a ring which is noncommutative left and right Noetherian, while $R[[x]]$ does not have right property (A). In this article, we study the above result of Hashemi et al. [9] to the skew Hurwitz series ring (HR, ω) . Here, we establish a relation between a ring with property (A) and its skew Hurwitz power series ring (HR, ω) , where ω is an endomorphism of R .

We need some standard definitions to understand the main and associated results of this article. A ring R is called (i) reduced if it has no nonzero nilpotent elements, (ii) symmetric if for all $a, b, c \in R$, $abc = 0$ implies $acb = 0$, (iii) reversible if $ab = 0$ implies $ba = 0$ for $a, b \in R$, (iv) semicommutative if for all $a, b \in R$, $ab = 0$ implies $aRb = 0$, (v) right (left) duo if every right (left) ideal is two-sided, (vi) abelian if all idempotents are central, (vii) biregular if every principal ideal of R is generated by central idempotents of R and NI if $\text{nil}(R)$ forms an ideal.

2. SKEW HURWITZ SERIES RINGS WITH PROPERTY (A)

Rings of formal power series have been interesting as they possess important applications. One of these is differential algebra [24]. Keigher [25] considered a variant of the ring of formal power series and studied some of its properties. In [26], he extended the study of this type of rings and introduced the ring of Hurwitz series over a commutative ring with identity. Moreover, he showed that the Hurwitz series ring HR is very closely connected to the base ring R itself if R is of positive characteristic. Recall the construction of Hurwitz series ring from [26], [27]. The elements of the Hurwitz series HR are sequences of the form $a = (a_n) = (a_1, a_2, a_3, \dots)$, where $a_n \in R$ for each $n \in \mathbb{N} \cup \{0\}$. Addition in HR is point-wise, while the multiplication of two elements (a_n) and (b_n) in HR is defined by $(a_n)(b_n) = (c_n)$, where

$$c_n = \sum_{k=0}^n C_k^n a_k b_{n-k}.$$

Here, C_k^n is a binomial symbol $n!/(k!(n-k)!)$ for all $n \geq k$, where $n, k \in \mathbb{N} \cup \{0\}$. This product is similar to the usual product of formal power series, except the binomial coefficients C_k^n . This type of product was considered first by Hurwitz [19], and then by Bochner and Martin [6], Fliess [8] and Taft [39] also. Inspired by the contribution of Hurwitz, Keigher [26] coined the term ring of Hurwitz series over commutative rings. After that, a number of authors, see for example [1], [5], [12], [13], [29], [30], [31], [34], [36], [37], [38], have studied the properties of abstract ring structures of the skew Hurwitz series ring (HR, ω) . Now, we see the construction of the skew Hurwitz series ring. Let R be a ring and $\omega: R \rightarrow R$ be an endomorphism of R , and $\omega(1) = 1$. The elements of (HR, ω) are functions $f: \mathbb{N} \cup \{0\} \rightarrow R$. Addition in (HR, ω) is component-wise. Multiplication is defined for every $f, g \in (HR, \omega)$ by

$$fg(p) = \sum_{k=0}^p C_k^p f(k) \omega^k(g(p-k))$$

for all $p, k \in \mathbb{N} \cup \{0\}$.

It can be easily shown that (HR, ω) is a ring with identity h_1 , defined by

$$h_1(n) = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{if } n \neq 0, \end{cases}$$

where $n \in \mathbb{N} \cup \{0\}$. It is clear that R is canonically embedded as a subring of (HR, ω) via $a \rightarrow h_a \in (HR, \omega)$, where

$$h_a(n) = \begin{cases} a & \text{if } n = 0, \\ 0 & \text{if } n \geq 1. \end{cases}$$

For any function $f \in (HR, \omega)$, $\text{supp}(f) = \{n \in \mathbb{N} \cup \{0\}; f(n) \neq 0\}$ denotes the support of f and $\pi(f)$ denotes the minimal element of $\text{supp}(f)$. For any nonempty subset X of R , we denote:

$$(HX, \omega) = \{f \in (HR, \omega); f(n) \in X \cup \{0\}, n \in \mathbb{N} \cup \{0\}\}.$$

Notice that if we take a skew formal power series $f(x) = \sum_{i=0}^{\infty} a_i x^i \in R[[x; \omega]]$ with the function $f(n) = a_n$, then the multiplication in skew Hurwitz series (HR, ω) is similar to the usual product of skew formal power series, except that binomial coefficients appear in each term of elements of (HR, ω) .

Due to Krempe [27], a monomorphism ω of a ring R is said to be rigid if $a\omega(a) = 0$ implies $a = 0$ for all $a \in R$. A ring R is called ω -rigid if there exists a rigid endomorphism ω of R . Annin [3] called a ring R to be ω -compatible if for every $a, b \in R$, $ab = 0$ if and only if $a\omega(b) = 0$. Hashemi and Moussavi [11] gave some examples of nonrigid ω -compatible rings. They proved the following lemma.

Lemma 2.1. *Let ω be an endomorphism of a ring R . Then*

- (1) *if ω is compatible, then ω is injective,*
- (2) *ω is compatible if and only if for all $a, b \in R$, $\omega(a)b = 0 \Leftrightarrow ab = 0$,*
- (3) *the following conditions are equivalent:*
 - (a) *ω is rigid,*
 - (b) *ω is compatible and R is reduced,*
 - (c) *for every $a \in R$, $\omega(a)a = 0$ implies that $a = 0$.*

To prove the main result we need to prove the following proposition.

Proposition 2.2. *Let R be a right duo and right Noetherian ring which is ω -compatible and torsion-free as a (Z) -module. If for any f and $g \in (HR, \omega)$, $fg = 0$, there exists $r \in R$ such that $f(m)g(n)r = 0$ for all $m, n \in \mathbb{N} \cup \{0\}$ and $g(n)r \neq 0$.*

Proof. Let $f, g \in (HR, \omega)$ such that $fg = 0$. Then we have

$$(2.1a) \quad f(0)g(0) = 0,$$

$$(2.1b) \quad f(0)g(1) + f(1)\omega(g(0)) = 0,$$

$$(2.1c) \quad f(0)g(2) + 2f(1)\omega(g(1)) + f(2)\omega^2(g(0)) = 0,$$

$$(2.1d) \quad \vdots$$

From (2.1a) we have $f(0)g(0) = 0$. It follows that $f(0)R\omega(g(0)) = 0$ since R is ω -compatible and semicommutative. Now, multiplying (2.1b) from left by $f(0)$,

we have $(f(0))^2g(1) = 0$. Then $2(f(0))^2R\omega(g(1)) = 0$ since R is ω -compatible, semicommutative and torsion-free as a (Z) -module. Now, multiplying (2.1c) from left by $(f(0))^2$, we get $(f(0))^2g(2) = 0$. Continuing this, we obtain $(f(0))^{n+1}g(n) = 0$. Since $r.\text{ann}_R(f(0)) \subseteq r.\text{ann}_R\omega((f(0))) \subseteq r.\text{ann}_R(\omega^2(f(0))) \subseteq \dots$ and R is right Noetherian, then there exists $k > 0$ such that $r.\text{ann}_R(f(0)^k) = r.\text{ann}_R(f(0)^t)$ for all $t \geq k$. Therefore $f(0)^k g(n) = 0$ for all $n \in \mathbb{N} \cup \{0\}$. Suppose $k > 0$ is the smallest positive integer such that $f(0)^k g(n) = 0$ for all $n \in \mathbb{N} \cup \{0\}$, $f(0)^k F = 0$, where $F = \{g(0), g(1), g(2), \dots, g(n)\}$. Then from [9], Lemma 2.6 there exists $r_0 \in R$ such that $f(0)g(n)r_0 = 0$ but $g(n)r_0 \neq 0$ for all $n \in \mathbb{N} \cup \{0\}$. Now from equations (2.1a), (2.1b), (2.1c), \dots and ω -compatibility of R , we have:

$$\begin{aligned} (2.2a) \quad & f(1)\omega(g(0))r_0 = 0, \\ (2.2b) \quad & 2f(1)\omega(g(1))r_0 + f(2)\omega^2(g(0))r_0 = 0, \\ (2.2c) \quad & 3f(1)\omega(g(2))r_0 + 3f(2)\omega^2(g(1))r_0 + f(3)\omega^3(g(0))r_0 = 0, \\ (2.2d) \quad & \vdots \end{aligned}$$

Applying the same logic and [9], Lemma 2.6, we obtain that there exists $r_1 \in R$ such that $f(1)g(n)r_0r_1 = 0$ but $g(n)r_0r_1 \neq 0$ for all $n \in \mathbb{N} \cup \{0\}$. Continuing this process we get $r_0, r_1, r_2, \dots, r_m \in R$ such that $f(m)g(n)r_0r_1r_2 \dots r_m = 0$ but $g(n)r_0r_1r_2 \dots r_m \neq 0$ for all $m, n \in \mathbb{N} \cup \{0\}$. Thus, there exists $r = r_0r_1, r_2, \dots, r_m \in R$ such that $f(m)g(n)r$ but $g(n)r \neq 0$ for all $m, n \in \mathbb{N} \cup \{0\}$. \square

Now, we prove the main result.

Theorem 2.3. *Let R be an ω -compatible ring which is torsion-free as a \mathbb{Z} -module. If R is right duo right Noetherian, then (HR, ω) has right property (A).*

Proof. Let $J = \langle f_1, f_2, \dots, f_n \rangle$ be a finitely generated two-sided ideal of (HR, ω) such that $J \subseteq Z_l((HR, \omega))$. Consider $I = \langle \bigcup_{i=1}^n C_{f_i} \rangle$, where C_{f_i} is a set of all the coefficients of f_i for all $1 \leq i \leq n$. Since $J \subseteq Z_l((HR, \omega))$, for some $g \in (HR, \omega)$, $f_i g = 0$ for all $1 \leq i \leq n$. Thus, from Proposition 2.2 there exists $r \in R$ such that $f_i(p)g(q)r = 0$ but $g(q)r \neq 0$ for all $p, q \in \mathbb{N} \cup \{0\}$ and $1 \leq i \leq n$. Thus, $Ig(q)r = 0$ but $g(q)r \neq 0$ for all $q \in \mathbb{N} \cup \{0\}$. Since R is semi-commutative, so I is an ideal of R and $I \subseteq Z_l(R)$. From [9], Remark 2.3, $Z_l(R) = \cup P_i$, where P_i is completely prime ideal and $p_i = l.\text{ann}_R(c_i)$ for a nonzero $c_i \in R$. Thus, from [9], Lemma 2.4 $I \subseteq P_i$ for some i . Therefore $Ic_i = 0$. It follows that $Jh_{c_i} = 0$, where h_{c_i} is a nonzero element of (HR, ω) . Hence, skew Hurwitz series ring (HR, ω) has right property (A). \square

Corollary 2.4. *Let R be a ring which is torsion-free as a \mathbb{Z} -module. If R is right duo right Noetherian, then (HR, ω) has right property (A).*

Proof. Let ω be an identity endomorphism of R , then $(HR, \omega) \cong (HR)$. Since R is right duo right Noetherian, from Theorem 2.3, (HR) has right property (A). \square

In [21] Jacobson stated that a right ideal of R is bounded if it contains a nonzero ideal of R . Further, Faith [7] generalized this concept and said that a ring R is strongly right (or left) bounded if every nonzero right (or left) ideal is bounded. A ring R is said to be strongly bounded if it is both strongly right bounded and strongly left bounded. After that, Hwang et al. [20] introduced the concept of a strongly right AB ring, which is a generalization of strongly bounded rings and semicommutative rings. A ring R is called strongly right (or left) AB if every nonzero right (or left) annihilator is bounded.

Theorem 2.5. *Let R be an ω -compatible ring which is torsion-free as a \mathbb{Z} -module. If R is right duo right Noetherian, then (HR, ω) is a strongly AB ring.*

Proof. Let A be a nonzero subset of (HR, ω) with $r.\text{ann}_{(HR, \omega)}(A) \neq 0$ and C_g be a set of coefficients of all $g \in A$. Then for a nonzero $f \in r.\text{ann}_{(HR, \omega)}(A)$, $gf = 0$. Since R is right duo right Noetherian and ω -compatible, so from Proposition 2.2 there exists a nonzero $r_0 \in R$ such that $g(n)f(m)r_0 = 0$ with $f(m)r_0 \neq 0$, for all $m, n \in \mathbb{N} \cup \{0\}$. It follows that $r.\text{ann}_R(C_g) \neq 0$ for all $g \in A$. Therefore there exists a nonzero ideal I such that $I \subseteq r.\text{ann}_R(C_g)$ since R is a strongly right AB ring. Thus, $(HI, \omega) \subseteq r.\text{ann}_{(HR, \omega)}(A)$. This implies that $r.\text{ann}_{(HR, \omega)}(A)$ contains a nonzero ideal (HI, ω) . Hence, (HR, ω) is strongly right AB. \square

In [1], Ahmadi et al. introduced the concept of skew Hurwitz series-wise Armendariz by considering R as a commutative ring, defined as follows:

Definition 2.6. Let R be a commutative ring and $\omega: R \rightarrow R$ be an endomorphism of R . The ring R is said to be skew Hurwitz series-wise Armendariz if for every skew Hurwitz series $f, g \in (HR, \omega)$, $fg = 0$ if and only if $f(n)g(m) = 0$ for all n, m .

Sharma and Singh [37] gave the definition of skew Hurwitz series-wise Armendariz in case of noncommutative ring. For more details about Armendariz rings and their generalizations, see [2], [16], [33].

Definition 2.7. Let R be a ring and $\omega: R \rightarrow R$ be an endomorphism of R . The ring R is said to be skew Hurwitz series-wise Armendariz if for every skew Hurwitz series $f, g \in (HR, \omega)$, $fg = 0$ implies $f(n)\omega^n g(m) = 0$ for all n, m .

Theorem 2.8. *Let R be a ring which is skew Hurwitz series-wise Armendariz and ω -compatible. Then the following statements are equivalent:*

- (1) R is strongly right AB.
- (2) (HR, ω) is strongly right AB.

Proof. (i) \rightarrow (ii) Let A be a nonzero subset of (HR, ω) with $r.\text{ann}_{(HR, \omega)}(A) \neq 0$ and let C_g be a set of coefficients of all $g \in A$. Then for a nonzero $f \in r.\text{ann}_{(HR, \omega)}(A)$, $gf = 0$. Therefore $g(n)f(m) = 0$ for all $m, n \in \mathbb{N} \cup \{0\}$ since R is skew Hurwitz series-wise Armendariz and ω -compatible. It follows that $r.\text{ann}_R(C_g) \neq 0$ for all $g \in A$. Therefore there exists a nonzero ideal I such that $I \subseteq r.\text{ann}_R(C_g)$ since R is a strongly right AB ring. Thus, $(HI, \omega) \subseteq r.\text{ann}_{(HR, \omega)}(A)$. This implies that $r.\text{ann}_{(HR, \omega)}(A)$ contains a nonzero ideal (HI, ω) . Hence, (HR, ω) is strongly right AB.

(ii) \rightarrow (i) Suppose A is a nonzero subset of R with $r.\text{ann}_R(A) \neq 0$. And we know that $r.\text{ann}_R(A) = r.\text{ann}_{(HR, \omega)}(A) \cap A$. It follows that $r.\text{ann}_{(HR, \omega)}(A) \neq 0$. Since (HR, ω) is strongly right AB, there exists a nonzero ideal I of (HR, ω) such that $I \subseteq r.\text{ann}_{(HR, \omega)}(A)$. Now, suppose I_f is a set of coefficients of all $f \in I$. Then I_f is a nonzero ideal of R and $I_f \subseteq r.\text{ann}_{(HR, \omega)}(A)$. Thus, $I_f \subseteq r.\text{ann}_R(A)$. Hence, R is strongly right AB. \square

As a direct consequence of the above theorem, we obtain the following corollary.

Corollary 2.9. *Let R be a ring which is skew Hurwitz series-wise Armendariz and ω -compatible. Then the following statements are equivalent:*

- (1) R is strongly right AB.
- (2) HR is strongly right AB.

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